DISPLAY OF EXECUTIVE SUMMERY

OF

"MODULAR LATTICES AND TRELLICES"

MINOR RESEARCHPROJECT REPORT

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Boolean algebra plays an important role in lattice theory. They are generalization of a power set algebra or a field of sets .It is also a special case of De Morgan algebra and a Kleeno algebra(with involution).In the branch of mathematics called ordered theory a modular lattice is a lattice that satisfies the modular law.Modular lattices arise in Algebra and in many other areas of mathematics.All distributive lattices ,the lattice of normal subgroups of any group, the lattice of submodules of any module and the subspaces of vector space are some examples of modular lattices. Bounded distributive lattices and complemented lattices are generalization of Boolean algebra but both the concepts are independent in the sense that bounded distributive lattice need not be complemented &vice-versa.

Theorems of lattice theory,thus demonstrating the superfluity of the Boolean lattices or Boolean algebra may be described as the richest and at the same time ,the most important lattices for application.Boolean algebras are special lattices which are useful in modeling,in the study of logic ,both digital computer logic and that of human thinking and switching circuits or relay circuits .C. L. Shannon

showed that fundamental properties of electrical circuitcs of bistable elements can be represented by using Boolean algebras. Lattices also occur in other parts of mathematics in a natural way.

.The idea of transitiviry& partial order are fundamental in wide varity of mathematical theories.The mathematical underground,however,has been simmering for some times with notions of non-transitive relations,some arising from common,every-day observations and some from purely mathematical considerations. The cricket team provide an immediate example.It may be noted that team A beats team B and team B beats team C, but it is far from certain that team A will beat team C.The theory of graphs has long been considered with non-transitive relations.Though vertex A impinges on vertex B and B on C ,it is not necessarily true that nor even desired that , A impinges on C .Theorem about non-tansitive graph may be found throught the literature.We thus come across a new type of logic of non-transitive implication.

The important step in the theory of partial ordering was the postulation of least upper bound and greatest lower bound and the development of the theory of lattices.Transitivityu is necessary for the associativity of the operations of least upper bound and greatest lower bound. Associativity has been regared as essential to the theory of lattices as the properties of many theorems heavily depend upon it. So it would seen that transitivity is an indispensable requirement for lattice theory

.However , starting out with a reflexive and antisymmetric but not necessarily transitive order ,we can define least upper bounds and greatest lower bounds similarly as for partial order sets , thus obtaihing a structure called a trellis,in which these operations are not necessasarilyassociative.With this approach we can prove nearly all the basic assumption of associativity. However ,in the presence of certain assumtions such as distributivity,relative complementation and modularity or others ,associativity follows as a consequence.

We have introduced some properties of 0-distributive lattices and 1-distributive lattices. Also every 0-distributive lattice is 0- modular. A complemented distributive lattice is known as Boolean algebras or Boolean lattice.If L is complemented and 0- distributive lattice then tha following statements are equivalent. (i)L is 0-modular (ii) L is unicomplemented (iii) L is Boolean (iv) L is weakly complemented. Thus Boolean lattice is a 0-modular lattice,unicomplemented and weakly complemented. Trellices are weakly associative lattices.